ON S-SPLIT p-HILBERT CLASS FIELD TOWERS WITH PRESCRIBE GALOIS GROUP

Presented by : **Karim SANKARA**

Nazi Boni University of Bobo-Dioulasso (BURKINA-FASO)

Laboratoire d' **A**lgèbre, de **M**athématiques **D**iscrètes et d'**I**nformatique (**LAMDI**)

Supervisors: **Prof. Idrissa KABORE and Prof. Christian Maire**

Gabon Algebraic Days, 5th Edition

March 20, 2025

1.Background

1. Born in Bakro (Côte D'Ivoire)
2. in **2007**: Baccalaureate serie **C** in Lycée Ouezzin coulibaly (Bobo-Dioulasso)
3. **2008-2010**: **DEUG II + Bachelor degree**, of Mathematics in Polytechnic University of Bobo-Dioulasso which is currently called Nazi Boni University.
4. **February 2022** defense of my **Master of Mathematics** on

   Topic: Aspect algebrico-geometrique de quelques fonctions d'encodage utilisées en cryptographie.

   Supervisors:

   – Prof. Idrissa KABORE,from Nazi Boni University of Bobo-Dioulasso(Burkina-Faso)

   – Dr. Tony EZOME, Ecole Normale Supérieur de Libreville(Gabon)

2. About my PhD and its context

1. Since **September 2022**, I am registered at Nazi Boni university of Bobo-Dioulasso in **L**aboratoire d' **A**lgèbre, de **M**athématiques **D**iscrètes et d'**I**nformatique (**LAMDI**). It a PhD in codirection

   It is supervised by:

   – Prof. Idrissa KABORE from Nazi Boni University (Burkina-Faso)

   – Prof. Christian MAIRE from university Marie and Louis Pasteur.

2. It is a PhD in **Algebraic Number Theory**.

   The **topic** is :

   Inverse Galois Problem for Hilbert p-class Field Tower with Local Condition.

• In our works, we focus on recents works:

– **M. Ozaki** who proved then years ago the famous result :

For a number field K, we denote by $\mathbf{L}_p(K)$, its **p**-Hilbert class field tower.
It is also the maximal p-extension of K unramified everywhere.

Theorem (Ozaki (2011))

Given a finite **p**-group G, there exists a totaly imaginary number field **K** for which $Gal(L_p(K)/K) \simeq \mathbf{G}$.

Strategy of Ozaki's Theorem Proof

Ozaki start with a complex quadratique field K_0 such that :
– p is inert in $K_0/\mathbb{Q}$.
– $Cl_{K_0} = (1)$.

We notice that Ozaki does not estimate neither $[K : \mathbb{Q}]$ nor the the quantity of ramification in $K/\mathbb{Q}$

– **Hajir-Maire-Ramakrishna** have revisited the proof of Ozaki theorem, which allows them to relax the condition on the signature of K and to control the degree and ramifications of $K/\mathbb{Q}$. They actually proved that :

Theorem(Hajir-Maire-Ramakrishna(2022))

Given a finite **p**-group Γ, and a number field K_0 such that $Cl_{K_0} = (1)$, there exists infinitely extensions F/K_0 for which $Gal(L_p(F)/F) \simeq \Gamma$. And more:

1. if $\mu_p \not\subset K_0$, then $[F : K_0] = p^2 \#\Gamma$ and F/K_0 is tamely ramified with $\#\{\mathfrak{p} \quad ramified\} = 2 + log_p(\#\Gamma)$;
2. if $\mu_p \subset K_0$, then $[F : K_0] = p.(\#\Gamma)^2$ and F/K_0 is tamely ramified with $\#\{\mathfrak{p} \quad ramified\} = 1 + 3log_p(\#\Gamma)$.

In our works, we **generalize the result of Hajir-Maire-Ramakrishna**, by considering the p-Hilbert tours with specific local conditions.

3. Received Funds

Since 2023, I have been the recipient of a scholarship for a period of 3 years. It from Graduate Assistantships in Developing Countries (GRAID Program), International Mathematical Union (**IMU**).

I have Received from The international network of research of **CNRS** that is **AFRIMath**, fund for my travel : Fly tickets and train in 2023 for my stay in Besançon pour 3 month.

The institute **FEMTO-ST** has supported my train ticket and my various subway for my participation of the RJCAF 2024 in Paris at **IHP**.

In 2024 I am the recipient of a high-level scientific grant from the french Embassy in Burkina-Faso, for a 4-month stay in University of Marie and louis Pasteur, Besançon. September 1 to December 31, 2024.

4. Stays carried out as part of my thesis

1. **2024**. Institut FEMTO-ST, Université de Franche-Comté. Invitation of 4 month (September-December). Stay financed by French Embassy of Burkina-Faso.

   Rencontres des **J**eunes **C**hercheurs **A**fricains en **F**rance (**RJCAF**) in Paris at **Institut Henri Poincaré**, from 12 to 13 December, 2024.

2. **2023** . Institut FEMTO-ST, Université Marie et Louis Pasteur. Invitation of 4 month (February-May). Stay financed by Graduate School **EIPHI** and CNRS.

   – Conference COUNT 2023 at **CIRM, Luminy, Marseille**, February 27 to 03 March, 2023.

   – The 7^{th} mini symposium of the **Roman number theory association in Roma** (Italy), from 2 to 6 May, 2023

5. Result obtained and submited

Thanks to the funds, the stays allowed me to have great progress in my PhD works.

A join work with **Maire** allowed us to get a first result given by:

Theorem

Let K be a number field with a finite p-Hilbert tower $L_p(K)/K$; set $G := Gal(L_p(K)/K)$. Assume that $r_{K,1} + r_{K,2} \geq h^1_G + h^2_G$.
Let S be a finite set of primes of K.
Then there exists a tamely ramified extension F/K of degree p^m such that

1. $L_p(F) = L_p^S(F)$;
2. the Galois group $Gal(L_p^S(F)/F)$ is isomorphic to G;
3. the extension F/K is ramified at m primes;
4. $m \leq e_G$.

This result has been submitted.

Thank you for your attention

UNIVERSITÉ MARIE & LOUIS PASTEUR

femto-st SCIENCES & TECHNOLOGIES

AMBASSADE DE FRANCE AU BURKINA FASO
Liberté
Égalité
Fraternité